

2000 P1 A1	4C $\bullet^1 \sin p \cos q + \cos p \sin q$ $\bullet^2 10 \text{ and } 13$ $\bullet^3 \frac{8}{10} \cdot \frac{12}{13} + \frac{6}{10} \cdot \frac{5}{13}$ $\bullet^4 \frac{126}{130} \qquad \text{ans: } \frac{63}{65}$
02 P1 Q5	4C $\bullet^1 AC = \sqrt{2} \text{ and } BC = \sqrt{10}$ <i>stated or implied by</i> \bullet^3 $\bullet^2 \sin(a+b) = \sin a \cos b + \cos a \sin b$ $\bullet^3 \frac{1}{\sqrt{2}} \cdot \frac{3}{\sqrt{10}} + \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{10}}$ $\bullet^4 \frac{4}{\sqrt{20}} = \dots\dots = \frac{2}{\sqrt{5}}$
03 P1 Q10	5C, 1B $\bullet^1 \text{hypot} = \sqrt{80}$ $\bullet^2 \sin(p) = \frac{4}{\sqrt{80}} \text{ and } \cos(p) = \frac{8}{\sqrt{80}}$ $\bullet^3 \sin(2p) = 2 \sin(p) \cos(p)$ $\bullet^4 \sin(2p) = \frac{4}{5}$ $\bullet^5 \cos(2p) = \frac{3}{5} \qquad \bullet^6 \frac{4}{3}$
04 P1 Q10	7B $\bullet^1 D\hat{E}A = (2x^\circ + 90^\circ)$ $\bullet^2 \cos(2x^\circ) \cos(90^\circ) - \sin(2x^\circ) \sin(90^\circ)$ $\bullet^3 -\sin(2x^\circ)$ $\bullet^4 -2\sin(x^\circ) \cos(x^\circ)$ $\bullet^5 CE = \sqrt{1^2 + 3^2} = \sqrt{10} \qquad \text{stated}$ $\bullet^6 \sin(x^\circ) = \left(\frac{1}{\sqrt{10}}\right)$ <i>and</i> $\cos(x^\circ) = \frac{3}{\sqrt{10}}$ $\bullet^7 \cos D\hat{E}A = -2\left(\frac{1}{\sqrt{10}}\right)\left(\frac{3}{\sqrt{10}}\right) = -\frac{6}{10}$
05 P1 Q9	4C $\bullet^1 2\cos^2(x) - 1 = \frac{7}{25} \qquad \bullet^1 1 - 2\sin^2(x) = \frac{7}{25}$ $\bullet^2 \cos^2(x) = \frac{32}{50} \qquad \bullet^2 \sin^2(x) = \frac{18}{50}$ $\bullet^3 \cos(x) = \frac{4}{5} \qquad \bullet^3 \sin(x) = \frac{3}{5}$ $\bullet^4 \sin(x) = \frac{3}{5} \qquad \bullet^4 \cos(x) = \frac{4}{5}$ <p style="text-align: center;">OR</p>
05 P2 Q2	4C, 3C $\bullet^1 \cos(p) = \frac{8}{17}, \sin(p) = \frac{15}{17}$ $\bullet^2 \cos(q) = \frac{8}{10}, \sin(q) = \frac{6}{10}$ $\bullet^3 \sin(p) \cos(q) + \cos(p) \sin(q)$ $\bullet^4 \frac{15}{17} \times \frac{8}{10} + \frac{8}{17} \times \frac{6}{10} = \& \text{ complete}$

	<ul style="list-style-type: none"> •⁵ $\cos(p)\cos(q) - \sin(p)\sin(q)$ •⁶ $-\frac{13}{85}$ or equivalent fraction •⁷ $-\frac{84}{13}$ or equivalent fraction (eg $-\frac{7140}{1105}$)
06 P2 Q8	4C, 4B <ul style="list-style-type: none"> •¹ $\sin(a^\circ) = \frac{1}{\sqrt{5}}$ •² $\sin(2a^\circ) = 2\sin(a^\circ)\cos(a^\circ)$ •³ $\cos(a^\circ) = \frac{2}{\sqrt{5}}$ •⁴ $\sin(2a^\circ) = \frac{4}{5}$ •⁵ $\sin(3a^\circ) = \sin(2a^\circ)\cos(a^\circ) + \cos(2a^\circ)\sin(a^\circ)$ •⁶ $\cos(2a^\circ) = \frac{3}{5}$ •⁷ $\sin(3a^\circ) = \frac{4}{5} \cdot \frac{2}{\sqrt{5}} + \frac{3}{5} \cdot \frac{1}{\sqrt{5}}$ •⁸ $\sin(3a^\circ) = \frac{11}{5\sqrt{5}}$
07 P2 Q2	4C, 4C <ul style="list-style-type: none"> •¹ $\sqrt{5}$ and $\sqrt{10}$ s/i by •³ •² $\sin(c)\cos(d) + \cos(c)\sin(d)$ s/i by •³ •³ $\frac{1}{\sqrt{5}} \times \frac{3}{\sqrt{10}} + \frac{2}{\sqrt{5}} \times \frac{1}{\sqrt{10}}$ •⁴ $\frac{1}{\sqrt{2}}$ (accept any equivalent single fraction) •⁵ $2\sin(c)\cos(c)$ •⁶ $2 \times \frac{1}{\sqrt{5}} \times \frac{2}{\sqrt{5}} = \frac{4}{5}$ or equivalent •⁷ e.g. $\cos^2(d) - \sin^2(d)$ •⁸ $\frac{9}{10} - \frac{1}{10} = \frac{8}{10} = \frac{4}{5}$
09 P1 Q24	3C, 2C, 4C/B

	<p>•¹ $\sin \frac{\pi}{3} \cos \frac{\pi}{4} + \cos \frac{\pi}{3} \sin \frac{\pi}{4}$ s / i by •²</p> <p>•² $\frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} + \frac{1}{2} \times \frac{1}{\sqrt{2}}$</p> <p>•³ $\frac{\sqrt{3}+1}{2\sqrt{2}}$ or equivalent</p> <p>•⁴ $\sin A \cos B + \cos A \sin B + \dots$</p> <p>•⁵ $\dots + \sin A \cos B - \cos A \sin B$ and complete</p> <p>•⁶ $\frac{\pi}{12} = \frac{\pi}{3} - \frac{\pi}{4}$ stated explicitly Alternatives</p> <p>and A is $\frac{\pi}{3}$, B is $\frac{\pi}{4}$ s / i by •⁷ 1. for •⁶ to •⁸</p> <p>•⁷ $2 \sin \frac{\pi}{3} \cos \frac{\pi}{4}$ •⁶ $\sin\left(\frac{\pi}{12}\right) = \sin \frac{\pi}{3} \cos \frac{\pi}{4} - \cos \frac{\pi}{3} \sin \frac{\pi}{4}$</p> <p>•⁸ $2 \times \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}}$ •⁷ $\frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} - \frac{1}{2} \times \frac{1}{\sqrt{2}}$</p> <p>•⁹ $\frac{\sqrt{6}}{2}$ (accept $\sqrt{\frac{3}{2}}$ or $\frac{\sqrt{3}}{\sqrt{2}}$ but not $\frac{2\sqrt{3}}{2\sqrt{2}}$) •⁸ $\frac{\sqrt{3}-1}{2\sqrt{2}}$ or equivalent</p>
10 P1 Q23	<p>write in slope/intercept form •¹ $y = \frac{3}{2}x$ or $y = 1.5x$ stated explicitly</p> <p>connect gradient and $\tan a$ •² $m = \frac{3}{2}$ and $\tan a = \frac{3}{2}$ or $m = \tan a$ and $\tan a = \frac{3}{2}$</p> <p>calculate hypotenuse •³ $\sqrt{13}$ stated, or implied by •⁴</p> <p>state value of sine ratio •⁴ $\frac{3}{\sqrt{13}}$ or $\frac{3\sqrt{13}}{13}$ may not appear until (c)</p> <p>determine $\tan b$ •⁵ $\tan b = \frac{3}{4}$ stated, or implied by •⁶</p> <p>know to complete triangle •⁶ right angled triangle with 3 and 4 correctly shown</p> <p>determine hypotenuse •⁷ 5 stated, or implied by •⁸</p> <p>state values of sine and cos •⁸ $\sin b = \frac{3}{5}$ and $\cos b = \frac{4}{5}$ may not appear until (c)</p> <p>know to use addition formula •⁹ $\sin a \cos b - \cos a \sin b$</p> <p>substitute into expansion •¹⁰ $\frac{3}{\sqrt{13}} \times \frac{4}{5} - \frac{2}{\sqrt{13}} \times \frac{3}{5}$</p> <p>evaluate sine of compound angle •¹¹ $\frac{6}{5\sqrt{13}}$</p> <p>use $\sin(-x) = -\sin x$ •¹² $-\frac{6}{5\sqrt{13}}$</p>

PRE 2000 ANSWERS – Trig Formulae

1	<p>•¹ $\sin A = \frac{3}{5}$ and $\cos A = \frac{4}{5}$ •⁴ $\sin 2A \cos B + \cos 2A \sin B$</p> <p>•² $\sin 2A = 2 \times \frac{3}{5} \times \frac{4}{5} = \frac{24}{25}$ (accept 0.96) •⁵ $\sin B = \frac{5}{13}$ and $\cos B = \frac{12}{13}$ and $\frac{323}{325}$</p> <p>•³ $\cos 2A = e.g. \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2 = \frac{7}{25}$ (accept 0.28)</p>
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2	<ul style="list-style-type: none"> $\sin x^\circ \cos 30^\circ + \cos x^\circ \sin 30^\circ$ $\sin x^\circ = \frac{4}{5}$ & $\cos x^\circ = \frac{3}{5}$ $\frac{4}{5} \cdot \frac{\sqrt{3}}{2} + \frac{3}{5} \cdot \frac{1}{2}$ and completes proof
3	<ul style="list-style-type: none"> know to calculate missing sides $BD = 5, AD = \sqrt{24}$ $\cos x \cos y - \sin x \sin y = \frac{4}{5} \cdot \frac{5}{7} - \frac{3}{5} \cdot \frac{\sqrt{24}}{7}$
4	<div style="display: flex; justify-content: space-between;"> <div> <ul style="list-style-type: none"> sketch with $\frac{x}{2}$ marked in $r/a \Delta$ height of triangle $= \sqrt{5}$ $\sin x = 2 \sin \frac{1}{2}x \cos \frac{1}{2}x$ $\sin \frac{x}{2} = \frac{2}{3}$ and $\cos \frac{1}{2}x = \frac{\sqrt{5}}{3}$ $\sin x = \frac{4\sqrt{5}}{9}$ </div> <div style="text-align: center;">OR</div> <div> <ul style="list-style-type: none"> know to use cosine rule $\cos x = \frac{3^2 + 3^2 - 4^2}{2 \cdot 3 \cdot 3}$ $\frac{1}{9}$ draw $r/a \Delta$ or use $\cos^2 x + \sin^2 x = 1$ $\sin x = \frac{\sqrt{80}}{9}$ </div> </div>
5	<ul style="list-style-type: none"> "third side" $= \sqrt{20}$ $\sin \alpha = \frac{\sqrt{11}}{\sqrt{20}}$ or $\cos \alpha = \frac{3}{\sqrt{20}}$ $2 \times \frac{\sqrt{11}}{\sqrt{20}} \times \frac{3}{\sqrt{20}}$
6	<ul style="list-style-type: none"> $\sin \theta \cos 120 + \cos \theta \sin 120$ and $\cos \theta \cos 150 - \sin \theta \sin 150$ correct use of exact values simplification to zero
7	<div style="display: flex; justify-content: space-between;"> <div> <ul style="list-style-type: none"> $\sin \theta = \frac{3}{5}$ $\frac{24}{25}$ </div> <div> <ul style="list-style-type: none"> $2 \sin 2\theta \cos 2\theta$ $\cos 2\theta = \frac{7}{25}$ $\frac{336}{625}$ </div> </div>
8	<ul style="list-style-type: none"> $\cos P = \frac{5}{13}$ $\cos Q = \frac{4}{5}$ $\frac{12}{13} \times \frac{4}{5} + \frac{5}{13} \times \frac{3}{5}$
9	<ul style="list-style-type: none"> strat for \cos: eg $\cos^2 = 1 - \sin^2$ $\cos A = \frac{\sqrt{7}}{4}$ $\sin 2A = \frac{3\sqrt{7}}{8}$
10	<ul style="list-style-type: none"> strat for exact value: e.g. $\sin^2 D = 1 - \cos^2 D$ $\sin D = \frac{1}{\sqrt{5}}$ $\cos 2D = \frac{3}{5}$
11	<ul style="list-style-type: none"> $OA = OB = \sqrt{a^2 + b^2}$ $\sin \hat{AOB} = \sin(\hat{BOx} - \hat{AOx}) = \sin \hat{BOx} \cos \hat{AOx} - \cos \hat{BOx} \sin \hat{AOx}$ $\frac{a}{\sqrt{a^2 + b^2}} \cdot \frac{a}{\sqrt{a^2 + b^2}} - \frac{b}{\sqrt{a^2 + b^2}} \cdot \frac{b}{\sqrt{a^2 + b^2}}$ $\frac{a^2 - b^2}{a^2 + b^2}$

12	<ul style="list-style-type: none"> •¹ $\cos x = \frac{2}{\sqrt{11}}$ or $\sin x = \frac{\sqrt{7}}{\sqrt{11}}$ •² $\cos 2x = 2 \times \left(\frac{2}{\sqrt{11}}\right)^2 - 1$ •³ $-\frac{3}{11}$
13	<p>(a) •¹ proof e.g. showing rt - angled triangle with "1" and a°</p> <p>(b) •² Q is $(\cos(a - 45)^\circ, \sin(a - 45)^\circ)$</p> <p>(c) •³ R is $(\cos(a + 45)^\circ, \sin(a + 45)^\circ)$</p> <p>(d) •⁴ $\frac{\sin(a+45) - \sin(a-45)}{\cos(a+45) - \cos(a-45)}$</p> <p>•⁵ $\frac{\sin a \cos 45 + \cos a \sin 45 - \sin a \cos 45 + \cos a \sin 45}{\cos a \cos 45 - \sin a \sin 45 - \cos a \cos 45 - \sin a \sin 45}$</p> <p>•⁶ $\frac{2 \cos a \sin 45}{-2 \sin a \sin 45}$</p> <p>•⁷ $-\frac{1}{\tan a}$</p> <p>(e) •⁸ $m_{OP} = \frac{\sin a}{\cos a} = \tan a$</p> <p>•⁹ $m_{tgt \text{ at } P} = -\frac{1}{\tan a}$</p>
14	<p>(a) •¹ $CA = \frac{d}{\tan a}$</p> <p>•² $CB = \frac{d}{\tan(180-b)}$</p> <p>(b) •³ $AB = \frac{d}{\frac{\sin a}{\cos a}} - \frac{d}{\frac{\sin b}{\cos b}}$</p> <p>•⁴ $\frac{d \cos a}{\sin a} - \frac{d \cos b}{\sin b}$</p> <p>•⁵ $\frac{d \sin b \cos a - d \cos b \sin a}{\sin a \sin b}$</p> <p>(c) •⁶ $AB = 14$</p> <p>•⁷ 1.577 or 0.634 (comes from $AB = 1.577d$ or $d = 0.634 AB$)</p> <p>•⁸ 8.9 miles</p>